Problem I. Consider the following rank one matrix:

\[
A = \begin{bmatrix}
1 \\
-1
\end{bmatrix} \begin{bmatrix}
1 & -1 & 2
\end{bmatrix}.
\]

1. Find a basis for the null space \( N(A) \).
2. Using the Gram-Schmidt process construct an orthonormal basis for \( N(A) \).

Problem II. Consider the matrix

\[
A = \begin{bmatrix}
\cos(\theta) & 1 \\
0 & 0 \\
\sin(\theta) & 0
\end{bmatrix}.
\]

1. Apply the Gram-Schmidt process to find an orthonormal basis, \( q_1 \) and \( q_2 \), for the column space \( C(A) = \langle q_1, q_2 \rangle \).
2. Let the matrix \( Q = [q_1, q_2] \). Compute the matrix \( R = Q^T A \).
3. Show that \( A = QR \). Explain why.

Problem III. Solve the system of equations \( Ax = b \) without computing \( A \) where \( A = QR \), and \( Q \), \( R \), and \( b \) are given by:

\[
Q = \frac{1}{3} \begin{bmatrix}
1 & -2 \\
2 & -1 \\
2 & 2
\end{bmatrix}, \quad R = \frac{1}{3} \begin{bmatrix}
1 & -1 \\
0 & 1
\end{bmatrix}, \quad b = \begin{bmatrix}
1 \\
-1 \\
0
\end{bmatrix}
\]

Problem IV. Consider an \( m \)-vector \( u \) that has unit norm, \( u^T u = 1 \). Let the matrix \( Q = I - 2uu^T \).

1. What is the dimension of \( \langle u \rangle^\perp ? \)
2. Show that \( Qu = -u \).
3. Assume \( u \) is orthogonal to a nonzero \( m \)-vector \( v \). What is \( Qv ? \)