2. (i) By (2.11),

$$a_0 = \frac{1}{2}\int_0^2 f(x)\,dx = \frac{1}{2}\left\{\int_0^1 dx + \int_1^2 -dx\right\} = 0,$$

$$a_n = \int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx = \int_0^1 \cos\frac{n\pi x}{2}\,dx + \int_1^2 -\cos\frac{n\pi x}{2}\,dx$$

$$= \frac{2}{n\pi}\left[\sin\frac{n\pi x}{2}\right]_0^1 - \frac{2}{n\pi}\left[\sin\frac{n\pi x}{2}\right]_1^2 = \frac{4}{n\pi}\sin\frac{n\pi}{2}, \quad n = 1, 2, \ldots$$

$$\Rightarrow \quad f(x) \sim \sum_{n=1}^{\infty} \frac{4}{n\pi}\sin\frac{n\pi}{2}\,\cos\frac{n\pi x}{2},$$

$$\text{(series)} = \begin{cases} 1, & 0 \le x < 1, \\ 0, & x = 1, \\ -1, & 1 < x \le 2. \end{cases}$$

(ii) As in (i),

$$a_0 = \frac{1}{2}\int_0^2 f(x)\,dx = \frac{1}{2}\left\{\int_0^1 x\,dx + \int_1^2 -2\,dx\right\} = \tfrac{1}{2}\left\{\left[\tfrac{1}{2}\,x^2\right]_0^1 - 2\right\} = -\tfrac{3}{4},$$

$$a_n = \int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx = \int_0^1 x\cos\frac{n\pi x}{2}\,dx + \int_1^2 -2\cos\frac{n\pi x}{2}\,dx$$

$$= \left[x\,\frac{2}{n\pi}\,\sin\frac{n\pi x}{2}\right]_0^1 - \int_0^1 \frac{2}{n\pi}\,\sin\frac{n\pi x}{2}\,dx - \frac{4}{n\pi}\left[\sin\frac{n\pi x}{2}\right]_1^2$$

$$= \frac{2}{n\pi}\sin\frac{n\pi}{2} + \frac{4}{n^2\pi^2}\left[\cos\frac{n\pi x}{2}\right]_0^1 + \frac{4}{n\pi}\,\sin\frac{n\pi}{2}$$

$$= \frac{6}{n\pi}\,\sin\frac{n\pi}{2} + \frac{4}{n^2\pi^2}\,\cos\frac{n\pi}{2} - \frac{4}{n^2\pi^2}, \quad n = 1, 2, \ldots$$

$$\Rightarrow \quad f(x) \sim -\frac{3}{4} + \sum_{n=1}^{\infty}\left(\frac{6}{n\pi}\,\sin\frac{n\pi}{2} + \frac{4}{n^2\pi^2}\,\cos\frac{n\pi}{2} - \frac{4}{n^2\pi^2}\right)\cos\frac{n\pi x}{2},$$

$$(\text{series}) = \begin{cases} x, & 0 \le x < 1, \\ -\frac{1}{2}, & x = 1, \\ -2, & 1 < x \le 2. \end{cases}$$

(iii) Here $L = 2$, so

$$a_0 = \frac{1}{2}\int_0^2 f(x)\,dx = \frac{1}{2}\left[\int_0^1 (2+x)\,dx + \int_1^2 (1-x)\,dx\right]$$

$$= \tfrac{1}{2}\left\{\left[2x + \tfrac{1}{2}\,x^2\right]_0^1 + \left[x - \tfrac{1}{2}\,x^2\right]_1^2\right\} = 1,$$

$$a_n = \int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx = \int_0^1 (2+x)\cos\frac{n\pi x}{2}\,dx + \int_1^2 (1-x)\cos\frac{n\pi x}{2}\,dx$$

$$= \left[(2+x)\,\frac{2}{n\pi}\,\sin\frac{n\pi x}{2}\right]_0^1 - \int_0^1 \frac{2}{n\pi}\,\sin\frac{n\pi x}{2}\,dx$$

$$+ \left[(1-x)\,\frac{2}{n\pi}\,\sin\frac{n\pi x}{2}\right]_1^2 - \int_1^2 \frac{2}{n\pi}\,\sin\frac{n\pi x}{2}\,(-\,dx)$$

$$= \frac{6}{n\pi}\,\sin\frac{n\pi}{2} + \frac{4}{n^2\pi^2}\left[\cos\frac{n\pi x}{2}\right]_0^1 - \frac{4}{n^2\pi^2}\left[\cos\frac{n\pi x}{2}\right]_1^2$$

$$= \frac{6}{n\pi}\,\sin\frac{n\pi}{2} + \frac{8}{n^2\pi^2}\,\cos\frac{n\pi}{2} - \frac{4}{n^2\pi^2} + (-1)^{n+1}\frac{4}{n^2\pi^2}, \quad n = 1, 2, \ldots$$

$$\Rightarrow \quad f(x) \sim 1 + \sum_{n=1}^{\infty}\left[\frac{6}{n\pi}\,\sin\frac{n\pi}{2} + \frac{8}{n^2\pi^2}\,\cos\frac{n\pi}{2} - \frac{4}{n^2\pi^2} + (-1)^{n+1}\frac{4}{n^2\pi^2}\right]\cos\frac{n\pi x}{2},$$

$$\text{series} = \begin{cases} 2+x, & 0 \le x < 1, \\ \frac{3}{2}, & x = 1, \\ 1-x, & 1 < x \le 2. \end{cases}$$

(iv) With $L = 3$, we have

$$a_0 = \frac{1}{3}\int_0^3 f(x)\,dx = \frac{1}{3}\left[\int_0^1 dx + \int_2^3 (x-3)\,dx\right]$$

$$= \tfrac{1}{3}\big\{\big[x\big]_0^1 + \big[\tfrac{1}{2}\,x^2 - 3x\big]_2^3\big\} = \tfrac{1}{6},$$

$$a_n = \frac{2}{3}\int_0^3 f(x)\,\cos\frac{n\pi x}{3}\,dx = \frac{2}{3}\left[\int_0^1 \cos\frac{n\pi x}{3}\,dx + \int_2^3 (x-3)\cos\frac{n\pi x}{3}\,dx\right]$$

$$= \frac{2}{3}\left\{\frac{3}{n\pi}\left[\sin\frac{n\pi x}{3}\right]_0^1 + \left[(x-3)\,\frac{3}{n\pi}\,\sin\frac{n\pi x}{3}\right]_2^3 - \int_2^3 \frac{3}{n\pi}\,\sin\frac{n\pi x}{3}\,dx\right\}$$

$$= \frac{2}{3}\left\{\frac{3}{n\pi}\,\sin\frac{n\pi}{3} + \frac{3}{n\pi}\,\sin\frac{2n\pi}{3} + \frac{9}{n^2\pi^2}\left[\cos\frac{n\pi x}{3}\right]_2^3\right\}$$

$$= \frac{2}{n\pi}\,\sin\frac{n\pi}{3} + \frac{2}{n\pi}\,\sin\frac{2n\pi}{3} + \frac{6}{n^2\pi^2}\,\cos(n\pi) - \frac{6}{n^2\pi^2}\,\cos\frac{2n\pi}{3}, \quad n = 1, 2, \ldots$$

$$\Rightarrow \quad f(x) \sim \frac{1}{6} + \sum_{n=1}^{\infty}\left[\frac{2}{n\pi}\left(\sin\frac{n\pi}{3} + \sin\frac{2n\pi}{3}\right) - \frac{6}{n^2\pi^2}\left(\cos\frac{2n\pi}{3} + (-1)^{n+1}\right)\right]\cos\frac{n\pi x}{3},$$

$$\text{series} = \begin{cases} 1, & 0 \le x < 1, \\ \frac{1}{2}, & x = 1, \\ 0, & 1 < x < 2, \\ -\frac{1}{2}, & x = 2, \\ x - 3, & 2 < x \le 3. \end{cases}$$