2. (i) By (2.10) with $L = 2$,

$$b_n = \int_0^2 f(x)\sin\frac{n\pi x}{2}\,dx = \int_0^1 \sin\frac{n\pi x}{2}\,dx + \int_1^2 -\sin\frac{n\pi x}{2}\,dx$$

$$= -\frac{2}{n\pi}\left[\cos\frac{n\pi x}{2}\right]_0^1 + \frac{2}{n\pi}\left[\cos\frac{n\pi x}{2}\right]_1^2$$

$$= \frac{2}{n\pi}\left[1 + (-1)^n - 2\cos\frac{n\pi}{2}\right], \quad n = 1, 2, \ldots$$

$$\Rightarrow \quad f(x) \sim \sum_{n=1}^{\infty} \frac{2}{n\pi}\left[1 + (-1)^n - 2\cos\frac{n\pi}{2}\right]\sin\frac{n\pi x}{2},$$

$$(\text{series}) = \begin{cases} 1, & 0 < x < 1, \\ -1, & 1 < x < 2, \\ 0, & x = 0, 1, 2. \end{cases}$$

(ii) As in (i),

$$b_n = \int_0^2 f(x)\sin\frac{n\pi x}{2}\,dx = \int_0^1 x\sin\frac{n\pi x}{2}\,dx + \int_1^2 -2\sin\frac{n\pi x}{2}\,dx$$

$$= \left[x\left(-\frac{2}{n\pi}\right)\cos\frac{n\pi x}{2}\right]_0^1 + \int_0^1 \frac{2}{n\pi}\cos\frac{n\pi x}{2}\,dx + \frac{4}{n\pi}\left[\cos\frac{n\pi x}{2}\right]_1^2$$

$$= -\frac{2}{n\pi}\cos\frac{n\pi}{2} + \frac{4}{n^2\pi^2}\left[\sin\frac{n\pi x}{2}\right]_0^1 + \frac{4}{n\pi}\left[\cos(n\pi) - \cos\frac{n\pi}{2}\right]$$

$$= -\frac{6}{n\pi}\cos\frac{n\pi}{2} + \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2} + (-1)^n\frac{4}{n\pi}, \quad n = 1, 2, \ldots$$

$$\Rightarrow \quad f(x) \sim \sum_{n=1}^{\infty}\left[-\frac{6}{n\pi}\cos\frac{n\pi}{2} + \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2} + (-1)^n\frac{4}{n\pi}\right]\sin\frac{n\pi x}{2},$$

$$(\text{series}) = \begin{cases} x, & 0 \le x < 1, \\ -\frac{1}{2}, & x = 1, \\ -2, & 1 < x < 2, \\ 0, & x = 2. \end{cases}$$

(iii) Here $L = 2$, so

$$b_n = \int_0^2 f(x)\sin\frac{n\pi x}{2}\,dx = \int_0^1 (2+x)\sin\frac{n\pi x}{2}\,dx + \int_1^2 (1-x)\sin\frac{n\pi x}{2}\,dx$$

$$= \left[(2+x)\left(-\frac{2}{n\pi}\right)\cos\frac{n\pi x}{2}\right]_0^1 + \int_0^1 \frac{2}{n\pi}\cos\frac{n\pi x}{2}\,dx$$

$$+ \left[(1-x)\left(-\frac{2}{n\pi}\right)\cos\frac{n\pi x}{2}\right]_1^2 + \int_1^2 \frac{2}{n\pi}\cos\frac{n\pi x}{2}\,(-dx)$$

$$= -\frac{6}{n\pi}\cos\frac{n\pi}{2} + \frac{4}{n\pi} + \frac{4}{n^2\pi^2}\left[\sin\frac{n\pi x}{2}\right]_0^1$$

$$+ \frac{2}{n\pi}\cos(n\pi) - \frac{4}{n^2\pi^2}\left[\sin\frac{n\pi x}{2}\right]_1^2$$

$$= -\frac{6}{n\pi}\cos\frac{n\pi}{2} + \frac{4}{n\pi} + \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2} + \frac{2}{n\pi}\cos(n\pi) + \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$= -\frac{6}{n\pi}\cos\frac{n\pi}{2} + \frac{4}{n\pi} + \frac{8}{n^2\pi^2}\sin\frac{n\pi}{2} + (-1)^n\frac{2}{n\pi}, \quad n = 1, 2, \ldots$$

$$\Rightarrow \quad f(x) \sim \sum_{n=1}^{\infty}\left[-\frac{6}{n\pi}\cos\frac{n\pi}{2} + \frac{8}{n^2\pi^2}\sin\frac{n\pi}{2} + \frac{4}{n\pi} + (-1)^n\frac{2}{n\pi}\right]\sin\frac{n\pi x}{2},$$

$$\text{series} = \begin{cases} 2+x, & 0 < x < 1, \\ 1-x, & 1 < x < 2, \\ 0, & x = 0,\ 2, \\ \frac{3}{2}, & x = 1. \end{cases}$$

(iv) With $L = 3$, we have

$$b_n = \frac{2}{3}\int_0^3 f(x)\sin\frac{n\pi x}{3}\,dx = \frac{2}{3}\left[\int_0^1 \sin\frac{n\pi x}{3}\,dx + \int_2^3 (x-3)\sin\frac{n\pi x}{3}\,dx\right]$$

$$= \frac{2}{3}\left\{-\frac{3}{n\pi}\left[\cos\frac{n\pi x}{3}\right]_0^1\right.$$

$$+\left[(x-3)\left(-\frac{3}{n\pi}\right)\cos\frac{n\pi x}{3}\right]_2^3+\int_2^3\frac{3}{n\pi}\,\cos\frac{n\pi x}{3}\,dx\Bigg\}$$

$$=\frac{2}{3}\Bigg\{-\frac{3}{n\pi}\,\cos\frac{n\pi}{3}+\frac{3}{n\pi}-\frac{3}{n\pi}\,\cos\frac{2n\pi}{3}+\frac{9}{n^2\pi^2}\,\left[\sin\frac{n\pi x}{3}\right]_2^3\Bigg\}$$

$$=-\frac{2}{n\pi}\,\cos\frac{n\pi}{3}+\frac{2}{n\pi}-\frac{2}{n\pi}\,\cos\frac{2n\pi}{3}-\frac{6}{n^2\pi^2}\,\sin\frac{2n\pi}{3},\quad n=1,2,\ldots$$

$$\Rightarrow\quad f(x)\sim\sum_{n=1}^{\infty}\left(\frac{2}{n\pi}-\frac{2}{n\pi}\,\cos\frac{n\pi}{3}-\frac{2}{n\pi}\,\cos\frac{2n\pi}{3}-\frac{6}{n^2\pi^2}\,\sin\frac{2n\pi}{3}\right)\sin\frac{n\pi x}{3},$$

$$\text{series}=\begin{cases}1, & 0<x<1,\\ \frac{1}{2}, & x=1,\\ 0, & 1<x<2,\ x=0,\ 3,\\ x-3, & 2<x<3,\\ -\frac{1}{2}, & x=2.\end{cases}$$