(iii) We have

$$
\begin{aligned}
& B^2 - 4AC = 64 - 64 = 0 \quad \Rightarrow \quad \text{parabolic equation,} \\
& \frac{dy}{dx} = \frac{8}{2} = 4 \quad \Rightarrow \quad y = 4x + C_1 \\
\Rightarrow \quad & r = y - 4x \ \text{ and, say, } \ s = y \\
\Rightarrow \quad & \bar{A} = \bar{B} = 0, \ \bar{C} = 16, \ \bar{D} = \bar{E} = 0, \ \bar{F} = 64, \ \bar{G} = 16, \quad u(x, y) = v(r, s) \\
\Rightarrow \quad & v_{ss} + 4v = 1 \\
\Rightarrow \quad & v(r, s) = \varphi(r)\cos(2s) + \psi(r)\sin(2s) + \tfrac{1}{4} \\
\Rightarrow \quad & u(x, y) = \varphi(y - 4x)\cos(2y) + \psi(y - 4x)\sin(2y) + \tfrac{1}{4}.
\end{aligned}
$$

(iv) As in (iii),

$$
\begin{aligned}
& B^2 - 4AC = 576 - 576 = 0 \quad \Rightarrow \quad \text{parabolic equation,} \\
& \frac{dy}{dx} = -\frac{24}{32} = -\frac{3}{4} \quad \Rightarrow \quad y = -\tfrac{3}{4}\,x + C_1 \\
\Rightarrow \quad & r = 3x + 4y \ \text{ and, say, } \ s = y \\
\Rightarrow \quad & \bar{A} = \bar{B} = 0, \ \bar{C} = 9, \ \bar{D} = \bar{F} = 0, \ \bar{E} = -27, \ \bar{G} = 9, \quad u(x, y) = v(r, s) \\
\Rightarrow \quad & v_{ss} - 3v_s = 1 \\
\Rightarrow \quad & v(r, s) = \varphi(r) + \psi(r)e^{3s} - \tfrac{1}{3}\,s \\
\Rightarrow \quad & u(x, y) = \varphi(3x + 4y) + \psi(3x + 4y)e^{3y} - \tfrac{1}{3}\,y.
\end{aligned}
$$

(v) Here

$$
\begin{aligned}
& B^2 - 4AC = 900 - 900 = 0 \quad \Rightarrow \quad \text{parabolic equation,} \\
& \frac{dy}{dx} = \frac{30}{50} = \frac{3}{5} \quad \Rightarrow \quad y = \tfrac{3}{5}\,x + C_1 \\
\Rightarrow \quad & r = 5y - 3x \ \text{ and, say, } \ s = y \\
\Rightarrow \quad & \bar{A} = \bar{B} = 0, \ \bar{C} = 9, \ \bar{D} = 0, \ \bar{E} = -27, \ \bar{F} = 18, \ \bar{G} = -18rs, \quad u(x, y) = v(r, s) \\
\Rightarrow \quad & v_{ss} - 3v_s + 2v = -2rs \\
\Rightarrow \quad & v(r, s) = \varphi(r)e^{s} + \psi(r)e^{2s} - rs - \tfrac{3}{2}\,r \\
\Rightarrow \quad & u(x, y) = \varphi(5y - 3x)e^{y} + \psi(5y - 3x)e^{2y} - \tfrac{1}{2}\,(2y + 3)(5y - 3x).
\end{aligned}
$$