7. (i) By (10.16), (10.15), and (2.5),

$$q(x,y) = -\sin(\pi x)\sin(4\pi y), \quad L = 1, \quad K = 1/2$$

$$\Rightarrow \quad G(x,y;\xi,\eta) = -8\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\sin(n\pi\xi)\sin(2m\pi\eta)}{(n\pi)^2+(2m\pi)^2}\,\sin(n\pi x)\sin(2m\pi y)$$

$$\Rightarrow \quad u(x,y) = \int_0^{1/2}\int_0^1 -8\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\sin(n\pi\xi)\sin(2m\pi\eta)}{\pi^2(n^2+4m^2)}\,\sin(n\pi x)\sin(2m\pi y)$$

$$\times\left[-\sin(\pi\xi)\sin(4\pi\eta)\right]d\xi\,d\eta$$

$$= 8\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\pi^2(n^2+4m^2)}\left[\int_0^1 \sin(n\pi\xi)\sin(\pi\xi)\,d\xi\right]$$

$$\times\left[\int_0^{1/2}\sin(2m\pi\eta)\sin(4\pi\eta)\,d\eta\right]\sin(n\pi x)\sin(2m\pi y)$$

$$= \frac{8}{\pi^2(1^2+4\cdot 2^2)}\,\frac{1}{2}\,\frac{1}{4}\,\sin(\pi x)\sin(4\pi y) = \frac{1}{17\pi^2}\,\sin(\pi x)\sin(4\pi y).$$

(ii) As in (i),

$$q(x,t) \equiv 1, \quad L = 2, \quad K = 1$$

$$\Rightarrow \quad G(x,y;\xi,\eta) = -2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\sin(n\pi\xi/2)\sin(m\pi\eta)}{(n\pi/2)^2+(m\pi)^2}\sin\frac{n\pi x}{2}\,\sin(m\pi y)$$

$$\Rightarrow \quad u(x,y) = \int_0^1\int_0^2 -8\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\sin(n\pi\xi/2)\sin(m\pi\eta)}{\pi^2(n^2+4m^2)}\,\sin\frac{n\pi x}{2}\,\sin(m\pi y)\,d\xi\,d\eta$$

$$= -8\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\pi^2(n^2+4m^2)}\left[\int_0^2 \sin\frac{n\pi\xi}{2}\,d\xi\right]$$

$$\times\left[\int_0^1 \sin(m\pi\eta)\,d\eta\right]\sin\frac{n\pi x}{2}\,\sin(m\pi y)$$

$$= -8\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\pi^2(n^2+4m^2)}\left(-\frac{2}{n\pi}\right)\left[\cos\frac{n\pi\xi}{2}\right]_0^2$$

$$\times\left(-\frac{1}{m\pi}\right)\big[\cos(m\pi\eta)\big]_0^1 \sin\frac{n\pi x}{2}\,\sin(m\pi y)$$

$$= -\frac{16}{\pi^4}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\big[(-1)^n-1\big]\big[(-1)^m-1\big]}{nm(n^2+4m^2)}\,\sin\frac{n\pi x}{2}\,\sin(m\pi y).$$