4. (i) We have

$$\begin{aligned}
&\mathcal{L}[u](x,s) = sU(x,s) + 2, \quad \mathcal{L}[t+2] = \frac{1}{s^2} + \frac{2}{s}, \quad \mathcal{L}[\cos(2t)] = \frac{s}{s^2+4} \\
\Rightarrow \quad & U''(x,s) - 9sU(x,s) = 18 - \frac{9}{s^2} - \frac{18}{s}, \\
& U(0,s) = \frac{s}{s^2+4}, \quad U(x,s) \text{ bounded as } x \to \infty \\
\Rightarrow \quad & U(x,s) = C_1 e^{3\sqrt{s}\,x} + C_2 e^{-3\sqrt{s}\,x} + \frac{1}{s^3} + \frac{2}{s^2} - \frac{2}{s} \\
& \qquad = \left(\frac{s}{s^2+4} - \frac{1}{s^3} - \frac{2}{s^2} + \frac{2}{s} \right) e^{-3\sqrt{s}\,x} + \frac{1}{s^3} + \frac{2}{s^2} - \frac{2}{s} \\
\Rightarrow \quad & u(x,t) = \mathcal{L}^{-1}[U](x,t) \\
& \qquad = \left[\cos(2t) - \tfrac{1}{2}\,t^2 - 2t + 2 \right] * \left[\frac{3x}{2\sqrt{\pi}}\, t^{-3/2}\, e^{-9x^2/(4t)} \right] + \tfrac{1}{2}\,t^2 + 2t - 2.
\end{aligned}$$

(ii) Here

$$\begin{aligned}
&\mathcal{L}[u](x,s) = sU(x,s) - 1, \quad \mathcal{L}[e^{t-x}] = \frac{1}{s-1}\, e^{-x}, \quad \mathcal{L}[t^2 + 2t] = \frac{2}{s^3} + \frac{2}{s^2} \\
\Rightarrow \quad & U''(x,s) - sU(x,s) = -1 - \frac{1}{s-1}\, e^{-x}, \\
& U(0,s) = \frac{2}{s^3} + \frac{2}{s^2}, \quad U(x,s) \text{ bounded as } x \to \infty \\
\Rightarrow \quad & U(x,s) = C_1 e^{\sqrt{s}\,x} + C_2 e^{-\sqrt{s}\,x} + \frac{1}{s} + \frac{1}{(s-1)^2}\, e^{-x} \\
& \qquad = \left[\frac{2}{s^3} + \frac{2}{s^2} - \frac{1}{s} - \frac{1}{(s-1)^2} \right] e^{-\sqrt{s}\,x} + \frac{1}{s} + \frac{1}{(s-1)^2}\, e^{-x} \\
\Rightarrow \quad & u(x,t) = \mathcal{L}^{-1}[U](x,t) \\
& \qquad = \left[t^2 + 2t - 1 - te^t \right] * \left[\frac{x}{2\sqrt{\pi}}\, t^{-3/2}\, e^{-x^2/(4t)} \right] + 1 + te^{t-x}.
\end{aligned}$$

(iii) As above,

$$\begin{aligned}
&\mathcal{L}[u](x,s) = sU(x,s) - 3, \quad \mathcal{L}[-t\cos x] = -\frac{1}{s^2}\, \cos x, \quad \mathcal{L}[te^{-t}] = \frac{1}{(s+1)^2} \\
\Rightarrow \quad & U''(x,s) - \frac{s}{4}\, U(x,s) = -\frac{3}{4} + \frac{1}{4s^2}\, \cos x,
\end{aligned}$$

$$
\begin{aligned}
& U(0,s) = \frac{1}{(s+1)^2}, \quad U(x,s) \text{ bounded as } x \to \infty \\
\Rightarrow\quad & U(x,s) = C_1 e^{(\sqrt{s}/2)x} + C_2 e^{-(\sqrt{s}/2)x} + \frac{3}{s} - \frac{1}{s^2(s+4)} \cos x \\
& \qquad = \left[\frac{1}{(s+1)^2} - \frac{49}{16s} + \frac{1}{4s^2} + \frac{1}{16(s+4)} \right] e^{-(\sqrt{s}/2)x} \\
& \qquad\qquad + \frac{3}{s} - \frac{1}{16}\left(-\frac{1}{s} + \frac{4}{s^2} + \frac{1}{s+4} \right) \cos x \\
\Rightarrow\quad & u(x,t) = \mathcal{L}^{-1}[U](x,t) \\
& \qquad = \left[te^{-t} - \tfrac{49}{16} + \tfrac{1}{4}\, t + \tfrac{1}{16}\, e^{-4t} \right] * \left[\frac{x}{4\sqrt{\pi}}\, t^{-3/2}\, e^{-x^2/(16t)} \right] \\
& \qquad\qquad + 3 - \tfrac{1}{16}(4t - 1 + e^{-4t}) \cos x.
\end{aligned}
$$