Chapter 9

1. (i) By definition,

$$\begin{aligned} F(s) = \mathcal{L}[f](s) &= \int_0^\infty f(t)e^{-st}\,dt = \int_0^\infty (2t-1)e^{-st}\,dt \\ &= \left[(2t-1)\left(-\frac{1}{s}\right)e^{-st}\right]_0^\infty + \int_0^\infty \frac{1}{s}\,e^{-st}\,2\,dt \\ &= -\frac{1}{s} - \frac{2}{s^2}\left[e^{-st}\right]_0^\infty = -\frac{1}{s} + \frac{2}{s^2}, \quad s > 0. \end{aligned}$$

(ii) In the same way,

$$F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}\,dt = \int_0^\infty e^{2t}\,e^{-st}\,dt = \int_0^\infty e^{(2-s)t}\,dt = \frac{1}{2-s}\left[e^{(2-s)t}\right]_0^\infty;$$

if $s > 2$, then $\lim\limits_{t\to\infty} e^{(2-s)t} = 0$, so

$$F(s) = \frac{1}{s-2}, \quad s > 2.$$

(iii) Similarly,

$$F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}\,dt = \int_0^1 e^{-st}\,dt = -\frac{1}{s}\left[e^{-st}\right]_0^1 = \frac{1}{s}\,(1-e^{-s}).$$

2. (i) Using Table A5, we have

$$\begin{aligned} &\mathcal{L}[\sin(3t)] = \frac{3}{s^2+9} \quad \Rightarrow \quad \mathcal{L}[e^{-t}\sin(3t)] = \frac{3}{(s+1)^2+9} = \frac{3}{s^2+2s+10}, \\ &\mathcal{L}[t^4] = \frac{24}{s^5} \quad \Rightarrow \quad \mathcal{L}[e^{-t}\sin(3t) - 3t^4] = \frac{3}{s^2+2s+10} - \frac{72}{s^5}. \end{aligned}$$

(ii) As in (i),

$$\mathcal{L}[e^{4t}\cos(2t) + 4(t-3)^3 H(t-3)] = \frac{s-4}{(s-4)^2+4} + 4e^{-3s}\cdot\frac{6}{s^4}$$
$$= \frac{s-4}{s^2-8s+20} + \frac{24}{s^4}\,e^{-3s}.$$

3. (i) Similarly, we have

$$\frac{2s+1}{s^2-2s+26} = \frac{2s+1}{(s-1)^2+25} = \frac{2(s-1)+3}{(s-1)^2+25}$$
$$= 2\,\frac{s-1}{(s-1)^2+25} + \frac{3}{5}\,\frac{5}{(s-1)^2+5^2}$$
$$\Rightarrow \quad \mathcal{L}^{-1}\left[\frac{2s+1}{s^2-2s+26}\right] = e^t\big[2\cos(5t) + \tfrac{3}{5}\,\sin(5t)\big].$$

(ii) Here

$$\frac{3s+2}{s^2+6s+25} - \frac{2s}{(s-1)^2}\,e^{-s} = \frac{3(s+3)-7}{(s+3)^2+16} - \frac{(2s-2)+2}{(s-1)^2}\,e^{-s}$$
$$= 3\,\frac{s+3}{(s+3)^2+4^2} - \frac{7}{4}\,\frac{4}{(s+3)^2+4^2} - \left[\frac{2}{s-1} + \frac{2}{(s-1)^2}\right]e^{-s}$$
$$\Rightarrow \quad \mathcal{L}^{-1}\left[\frac{3s+2}{s^2+6s+25} - \frac{2s}{(s-1)^2}\,e^{-s}\right]$$
$$= e^{-3t}\big[3\cos(4t) - \tfrac{7}{4}\,\sin(4t)\big] - \big[2e^{t-1} + 2(t-1)e^{t-1}\big]H(t-1)$$
$$= e^{-3t}\big[3\cos(4t) - \tfrac{7}{4}\,\sin(4t)\big] - 2te^{t-1}H(t-1).$$