4. (i) We have

$$\begin{aligned}
& r(x,t) = C_1(t)x + C_2(t) \\
\Rightarrow\quad & r(0,t) = C_2(t) = 2t^2 + t, \quad r(1,t) = C_1(t) + C_2(t) = t^2 - 1 \\
\Rightarrow\quad & C_1(t) = -t^2 - t - 1 \quad \Rightarrow \quad r(x,t) = -(t^2 + t + 1)x + 2t^2 + t \\
\Rightarrow\quad & v(x,t) = u(x,t) - r(x,t) = u(x,t) + (t^2 + t + 1)x - 2t^2 - t.
\end{aligned}$$

(ii) Here

$$\begin{aligned}
& r_x(x,t) = C_1(t)x + C_2(t) \\
\Rightarrow\quad & r_x(0,t) = C_2(t) = 3t^2 + t, \quad r_x(1,t) = C_1(t) + C_2(t) = 2t^2 - 1 \\
\Rightarrow\quad & C_1(t) = -t^2 - t - 1 \quad \Rightarrow \quad r_x(x,t) = -(t^2 + t + 1)x + 3t^2 + t \\
\Rightarrow\quad & r(x,t) = -\tfrac{1}{2}(t^2 + t + 1)x^2 + (3t^2 + t)x \\
\Rightarrow\quad & v(x,t) = u(x,t) + \tfrac{1}{2}(t^2 + t + 1)x^2 - (3t^2 + t)x.
\end{aligned}$$