9. (i) With $L=1$, $K=1$, and $c=1$, we have

$$\begin{aligned}
&a_{12}=1, \quad a_{nm}=0,\ m\neq 2, \quad \sqrt{\lambda_{21}}\,b_{21}=-2, \quad \sqrt{\lambda_{nm}}\,b_{nm}=0,\ m\neq 1,\\
&\lambda_{12}=\pi^2+4\pi^2=5\pi^2, \quad \lambda_{21}=4\pi^2+\pi^2=5\pi^2\\
\Rightarrow\quad & u(x,y,t)=a_{12}\sin(\pi x)\sin(2\pi y)\cos(\sqrt{\lambda_{12}}\,t)\\
&\qquad\qquad +b_{21}\sin(2\pi x)\sin(\pi y)\sin(\sqrt{\lambda_{21}}\,t)\\
&\qquad =\sin(\pi x)\sin(2\pi y)\cos(\sqrt{5}\,\pi t)\\
&\qquad\qquad -\frac{2}{\sqrt{5}\,\pi}\sin(2\pi x)\sin(\pi y)\sin(\sqrt{5}\,\pi t).
\end{aligned}$$

(ii) By (5.51)–(5.53) with $L=1$, $K=1$, and $c=1$,

$$\begin{aligned}
\lambda_{nm}&=n^2\pi^2+m^2\pi^2=(n^2+m^2)\pi^2,\\
a_{nm}&=4\int_0^1\int_0^1 \sin(n\pi x)\sin(m\pi y)\,dx\,dy=\left\{-\frac{1}{n\pi}\left[\cos(n\pi x)\right]_0^1\right\}\left\{-\frac{1}{m\pi}\left[\cos(m\pi y)\right]_0^1\right\}\\
&=4\,\frac{1}{n\pi}\left[1-\cos(n\pi)\right]\frac{1}{m\pi}\left[1-\cos(m\pi)\right]\\
&=\left[1-(-1)^n\right]\left[1-(-1)^m\right]\frac{4}{nm\pi^2}, \quad n,m=1,2,\dots,
\end{aligned}$$

$$\begin{aligned}
b_{nm}&=\frac{4}{\pi\sqrt{n^2+m^2}}\int_0^1\int_0^1 xy\sin(n\pi x)\sin(m\pi y)\,dx\,dy\\
&=\frac{4}{\pi\sqrt{n^2+m^2}}\left\{\left[x\left(-\frac{1}{n\pi}\right)\cos(n\pi x)\right]_0^1+\int_0^1\frac{1}{n\pi}\cos(n\pi x)\,dx\right\}\\
&\qquad\times\left\{\left[y\left(-\frac{1}{m\pi}\right)\cos(m\pi y)\right]_0^1+\int_0^1\frac{1}{m\pi}\cos(m\pi y)\,dy\right\}\\
&=\frac{4}{\pi\sqrt{n^2+m^2}}\left\{-\frac{1}{n\pi}\cos(n\pi)\right\}\left\{-\frac{1}{m\pi}\cos(m\pi)\right\}\\
&=(-1)^{n+m}\frac{4}{\pi^3 nm\sqrt{n^2+m^2}}, \quad n,m=1,2,\dots
\end{aligned}$$

$$
\begin{aligned}
\Rightarrow \quad u(x,y,t) = &\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\big[1-(-1)^n\big]\big[1-(-1)^m\big]\,\frac{4}{nm\pi^2} \\
&\qquad\qquad \times \sin(n\pi x)\sin(m\pi y)\cos(\sqrt{n^2+m^2}\,\pi t) \\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}(-1)^{n+m}\,\frac{4}{\pi^3 nm\sqrt{n^2+m^2}} \\
&\qquad\qquad \times \sin(n\pi x)\sin(m\pi y)\sin(\sqrt{n^2+m^2}\,\pi t) \\
= &\frac{4}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{nm}\bigg\{\big[1-(-1)^n\big]\big[1-(-1)^m\big]\cos(\sqrt{n^2+m^2}\,\pi t) \\
&\qquad + (-1)^{n+m}\,\frac{1}{\pi\sqrt{n^2+m^2}}\,\sin(\sqrt{n^2+m^2}\,\pi t)\bigg\} \\
&\qquad\qquad \times \sin(n\pi x)\sin(m\pi y).
\end{aligned}
$$